$\cdot \lim\limits_{x\to+\infty} \dfrac{f_n(x)}{x} = \lim\limits_{x\to+\infty} \left[\dfrac{(\text{Log}x)^n}{x}\right] = \lim\limits_{x\to+\infty} \left(\dfrac{\text{Log}x}{x^{\frac{1}{n}}}\right)^n$

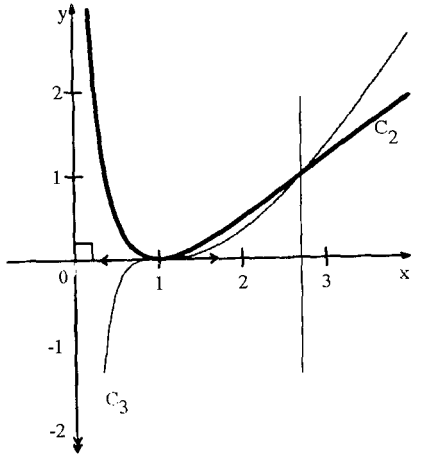
Posons $X = x^{\frac{1}{n}} \Leftrightarrow x = X^n;\ x \to +\infty; X \to +\infty$

$= \lim\limits_{X\to+\infty} \left[\dfrac{n\text{Log}X}{X}\right]^n = 0$

$\Rightarrow C_n$ possède une branche parabolique infinie de direction l'axe des abscisses au voisinage de $+\infty$.

x	0		1		$+\infty$
$f_2'(x)$	‖	$-$	0	$+$	
$f_2(x)$	‖ $+\infty$	↘	0	↗	$+\infty$

x	0		1		$+\infty$
$f_3'(x)$	‖	$+$	0	$+$	
$f_3(x)$	‖ $-\infty$	↗		↗	$+\infty$



3)a) $I_2 = \int_1^e (\text{Log}\,x)^2 dx$

$$\begin{pmatrix} u_1'(x) = 1 & \curvearrowright & u_1(x) = x \\ v_1(x) = (\text{Log}x)^2 & \curvearrowright & v_1'(x) = \frac{2}{x}\text{Log}x \end{pmatrix}$$

$= [x(\text{Log}x)^2]_1^e - 2\int_1^e \text{Log}x\,dx$

$$\begin{pmatrix} u_2'(x) = 1 & \curvearrowright & u_2(x) = x \\ v_2(x) = \text{Log}x & \curvearrowright & v_1'(x) = \frac{1}{x} \end{pmatrix}$$

$= e(\text{Log}e)^2 - 0 - 2\left([x\text{Log}x]_1^e - \int_1^e dx\right)$

$= e - 2e + 2(e-1) = e - 2.$

b) $I_{n+1} = \int_1^e (\text{Log}\,x)^{n+1} dx$

$$\begin{pmatrix} u'(x) = 1 & \curvearrowright & u(x) = x \\ v(x) = (\text{Log}x)^{n+1} & \curvearrowright & v'(x) = \frac{n+1}{x}(\text{Log}x)^n \end{pmatrix}$$

$I_{n+1} = [x(\text{Log}x)^{n+1}]_1^e - (n+1)\int_1^e (\text{Log}x)^n dx$

$I_{n+1} = e - (n+1)I_n.$

$\Leftrightarrow \underline{I_{n+1} + (n+1)I_n = e.}$